frequencies. This is a severely limiting restriction, as it assumes that elastance components reach their peak at the same instant.

Nelson [4] has derived a more realistic constraint based on 90° phase difference between the output charge and signal and pump charge, with the worst case phase condition between the latter two. It is given by

$$m_1 \cos\theta + \sqrt{m_2^2 + m_3^2 + 2m_2m_3 \sin\theta} \leq 0.25 \quad (1)$$

where

$$m_1 \sin\theta = \frac{m_2m_3 \cos\theta}{\sqrt{m_2^2 + m_3^2 + 2m_2m_3 \sin\theta}}. \quad (2)$$

The two equations are not sufficient to determine the elastance modulation ratios. They can, however, be determined if we introduce additional relations, based on Manley–Rowe equations, between them and the component frequencies, viz.,

$$m_1^2 f_1 = m_2^2 f_2 = m_3^2 f_3. \quad (3)$$

Eliminating the modulation ratios, we are left with a cubic equation for $y = \sin\theta$

$$y^3 + \sqrt{\frac{f_3}{f_2}}\, y^2 - \frac{1}{2} \frac{f_1}{\sqrt{f_2 f_3}} = 0 \quad (4)$$

which can be solved for given frequencies. Equation (1) can be rewritten as

$$m_1 = \frac{0.25}{\cos\theta + \sqrt{\frac{f_1}{f_2} + \frac{f_1}{f_3} + \frac{2f_1}{\sqrt{f_2 f_3}} \sin\theta}} \quad (5)$$

and m_1 can be calculated for the known value of θ. The remaining modulation ratios are then obtained from (3).

With the knowledge of modulation ratios, the input and output loading impedance can be determined:

$$R_{\text{in}} = R_s \left(1 + \frac{m_1 f_c}{\sqrt{f_2 f_3}}\right) \quad (6)$$

$$R_{\text{out}} = R_s \left(\frac{m_1 f_c}{\sqrt{f_2 f_3}} - 1\right) \quad (7)$$

where

R_s varactor series resistance,

f_c varactor cutoff frequency (at breakdown).

In this case f_c equals 600 GHz at -10 V and R_s equals 2.32 Ω. The frequencies were

$$f_1 = 2.4 \text{ GHz} \qquad f_2 = 12.7 \text{ GHz} \qquad f_3 = 15.1 \text{ GHz}.$$

Equations (1)–(3) yielded

$$m_1 = 0.1536 \qquad m_2 = 0.0668 \qquad m_3 = 0.0612$$

with

$$R_{\text{in}} = 20.2\,\Omega \qquad R_{\text{out}} = 15.6\,\Omega.$$

The following results were obtained: $P_{\text{out}} = 10$ mW at 15.1 GHz for $P_{\text{in}} = 4.7$ mW at 2.4 GHz and $P_{\text{in}} = 40$ mW at 12.7 GHz, with the pump efficiency of

$$\eta_p = \frac{10}{40} = 25 \text{ percent}$$

and signal gain

$$G = \frac{10}{4.7} = 3.8 \text{ dB}.$$

The theoretical varactor efficiency is given by

$$\eta_v = \frac{f_3}{f_2} \frac{m_1 f_c - \sqrt{f_2 f_3}}{m_1 f_c + \sqrt{f_2 f_3}} = 0.878.$$

Thus the circuit efficiency is

$$\eta_c = \frac{0.25}{0.878} = 0.285 \quad \text{or 5.4-dB loss.}$$

The loss in the two filters amounts to 2.6 dB, thus leaving 2.8 dB as the loss in the remaining circuit.

Reversing the pump and output terminals, the same circuit can be used as a lower-sideband upconverter. This is a negative-resistance circuit and is capable of much higher signal gain. The transducer gain is given by

$$G = \frac{R_g R_L m^2 f_c^2}{4 f_1^2 R_s^2 \left[\left(\frac{R_g}{R_s} + 1\right)\left(\frac{R_L}{R_s} + 1\right) - \frac{m^2 f_c^2}{f_1 f_2}\right]}.$$

In this case $m = m_3 = 0.0612$ and $R_g = R_L = R_{\text{in}} = 20.2$ Ω. Substituting, we obtain $G = 2.5$ dB.

It is important to note that this corresponds to the saturation value of the gain, as the signal and idler modulation ratios are greater than those of the pump. By decreasing the former, we can increase the pump-modulation ratio and the upconverter gain. This, of course, is achieved by lowering the input signal level and increasing the pump level. Under these conditions, transucucer gains of 13 dB and 100 MHz or 4-percent 3-dB signal bandwidth were measured.

References

[1] P. Bura, "Integrated S band upconverter," *RCA Eng.*, vol. 14, no. 5, pp. 86–87, Mar. 1969.
[2] P. Penfield and R. Rafuse, *Varactor Applications.* Cambridge, Mass.: M.I.T. Press, 1962.
[3] A. I. Grayzel, "A note on the abrupt junction large signal upconverter," *Proc. IEEE* (Lett.), vol. 54, pp. 78–79, Jan. 1966.
[4] C. E. Nelson and A. I. Grayzel, "A note on the large-signal varactor upper-sideband upconverter," *Proc. IEEE* (Lett.), vol. 54, pp. 1013–1014, July 1966.

Wide-Band Varactor-Tuned X-Band Gunn Oscillators in Full-Height Waveguide Cavity

J. S. JOSHI

***Abstract*—Some results on X-band varactor-tuned Gunn oscillators in a full-height waveguide cavity are presented. It is shown that it is possible to obtain an electronic tuning range of over 1 GHz at X band with an appreciable output power level, which is also nearly constant with frequency. Results of FM noise measurements on one such oscillator are also reported.**

Wide-band varactor-tuned Gunn oscillators have previously been reported in the literature [1]–[3]. Lee and Hodgart [1] obtained 1-GHz electronic tuning in *J* band, while Smith and Crane [2] obtained 1.1-GHz electronic tuning in *X* band. Downing and Myers [3] achieved 1.95 GHz of electronic tuning range in *X* band with a reduced-height waveguide cavity. These authors used inherently low-*Q* cavities in the form of either reduced-height waveguide or coaxial line structures. Previous attempts [3] to achieve a wide varactor tuning range in a full-height waveguide cavity at *X* band resulted in a maximum tuning of 200 MHz. However, by appropriately positioning the Gunn and varactor devices, an electronic tuning range in excess of 1 GHz has been realized here.

The experiments were conducted with Mullard Type CXY 19 Gunn devices and silicon tuning varactors. Both are encapsulated devices in the standard S4 package. Typical Gunn-device parameters are: $V_T = 4.75$ V, $I_T = 600$ mA, $V_G = 12.0$ V, $I_G = 450$ mA, $P_0 = 150$ mW in a standard test cavity. The devices were mounted on cylindrical post structures in a standard WG 16 waveguide block with suitable biasing arrangements incorporated. A moveable short circuit was used to form the waveguide cavity, and no matching elements

Manuscript received May 30, 1972; revised August 14, 1972.
The author is with Mullard Hazel Grove, School Street Hazel Grove, Stockport, Cheshire, England.

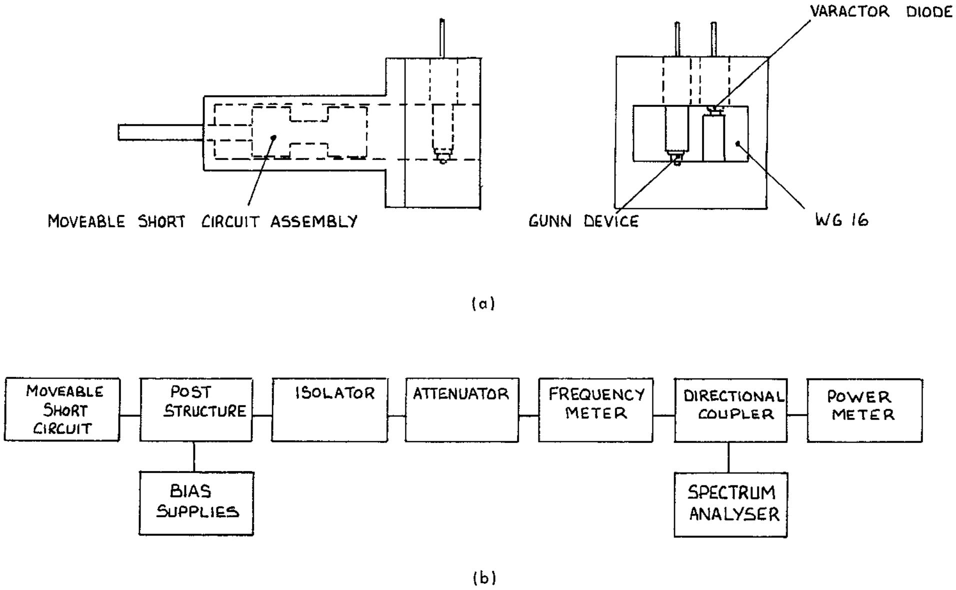

Fig. 1. (a) Post mounting configuration. (b) Experimental setup.

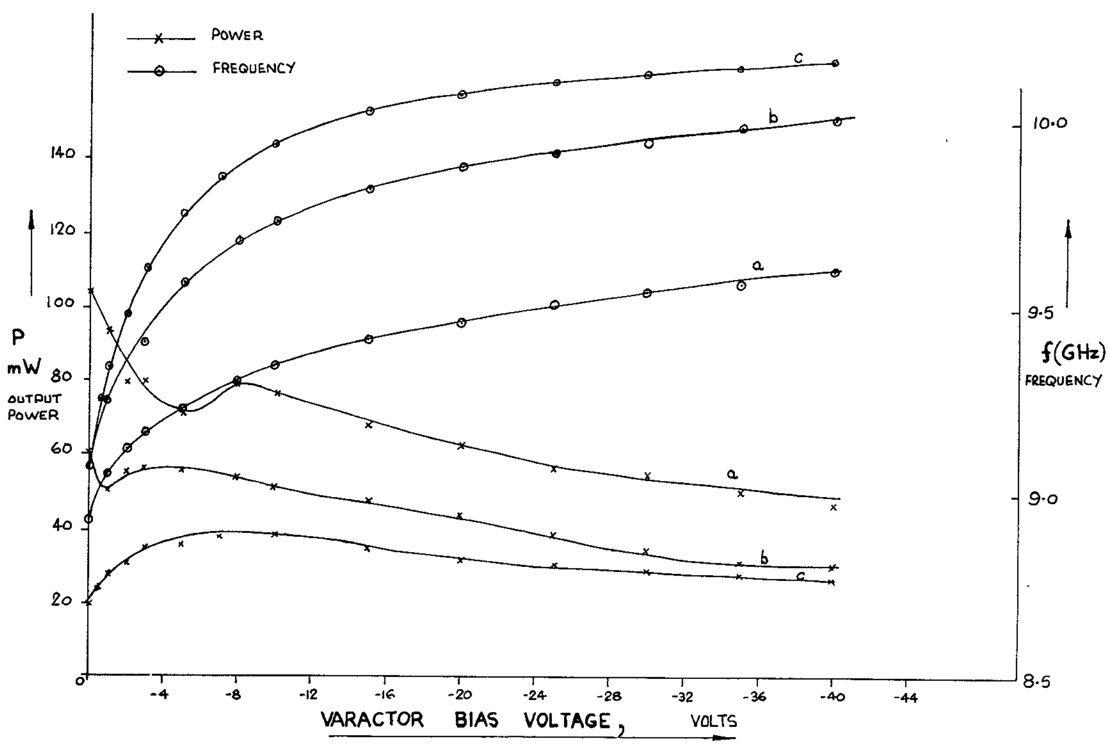

Fig. 2. Output power and frequency variation as a function of varactor bias. Gunn device CXY 19/3-5. Varactor diodes: *a*, *b*, and *c*.

were used. The diode post-structure plane was normal to the axis of the waveguide [Fig. 1(a)]. The post structures were brought together as near as possible in order to increase the coupling between the two devices. Location of the varactor diode was varied along the height of its post structure in order to further increase the degree of coupling. The maximum amount of coupling was achieved when the devices were at the opposite ends of the waveguide height, as shown in Fig. 1(a). The experimental setup used is as shown in Fig. 1(b). The spectrum analyzer was used to check on discontinuities, spurious operation, harmonics, etc.

Fig. 2 contains experimental results with Gunn device CXY 19/3-5 ($P_0=125$ mW in test cavity) and three different varactors with parameters as shown in the following table. These results show that as C_{j0} is reduced from 2.89 pF to 1.05 pF, the power is coupled more to the varactor. This results in an increase in the tuning range and a decrease in the output power. The maximum electronic tuning of 1.1 GHz at 20-mW minimum output power was achieved using a varactor with a C_{j0} of 1.05 pF. Further reduction of C_{j0} resulted in a reduced tuning range due to mode switching. Varations of the post geometry did not improve the tuning range. The slight drop in the output power for smaller values of varactor bias voltage is due to high interaction between the cavity and its tuning element. But in no case was the drop more than 3 dB in the output power level [3]. The self-bias voltage on the varactor diode terminals was between 0.5 and 0.7 V,

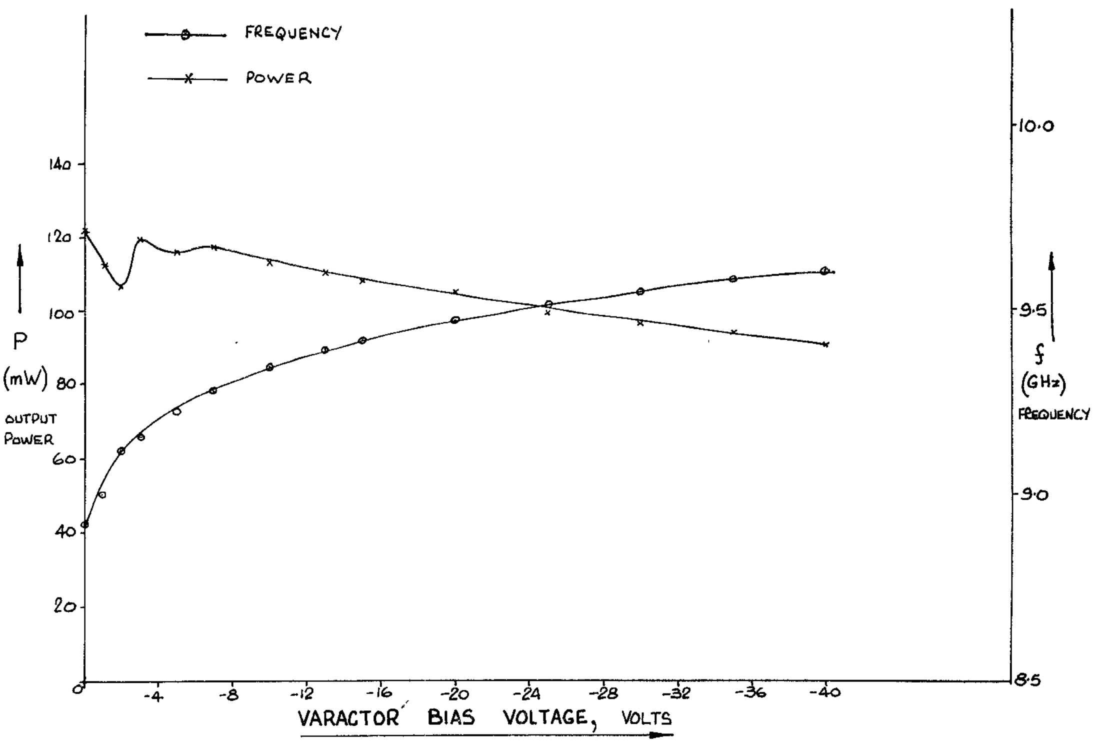

Fig. 3. Output power and frequency variation as a function of varactor bias. Gunn device CXY 19/1-2. Varactor diode: *a*.

and the rectified RF current through the varactors was not more than 50 μA. Following is a table of varactor diode parameters:

Type	C_{j0} (pF)	C_{j6} (pF)	V_{BR} (V)	V_F@1 mA (V)
a	2.89	1.05	50	0.78
b	1.67	0.59	50	0.76
c	1.05	0.39	52	0.76

The loaded Q of the cavities was estimated by frequency-pulling experiments. The variation in the values of Q_L was from 50 at zero varactor bias to 160 at −40 V varactor bias. These values for Q_L lie within the range obtainable in a single-post full-height waveguide cavity [4]. The FM noise of the oscillator of Fig. 2 *b* was measured and was found to be 65 rms Hz/$\sqrt{\text{Hz}}$ at 1-KHz off-carrier. The oscillator frequency was 9.95 GHz.

It has been demonstrated that a wide-band varactor-tuned Gunn oscillator can be constructed in standard X-band waveguide cavity. Furthermore, the oscillator constructed in full-height waveguide shows less variation in the output power over the tuning range and eliminates the requirement for a transition to X-band waveguide. With a higher power Gunn device (CXY 19/1-2, $P_0=170$ mW in test cavity) minimum power level of 90 mW with a variation of +1.3 dB and nearly 700 MHz of electronic tuning has been achieved (Fig. 3).

Acknowledgment

The author wishes to thank the members of the Semiconductor Measurements Laboratory for their help and co-operation, and J. A. F. Cornick for his continuous encouragement throughout this work.

References

[1] B. K. Lee and M. S. Hodgart, "Microwave Gunn oscillator tuned electronically over 1 GHz," *Electron. Lett.*, vol. 4, pp. 240–242, 1968.
[2] R. B. Smith and P. W. Crane, "Varactor-tuned Gunn-effect oscillator," *Electron. Lett.*, vol. 6, pp. 139–140, 1970.
[3] B. J. Downing and F. A. Myers, "Broadband (1.95 GHz) varactor-tuned X band Gunn oscillator," *Electron. Lett.*, vol. 7, pp. 407–409, 1971.
[4] W. C. Tsai, F. J. Rosenbaum, and L. A. MacKenzie, "Circuit analysis of waveguide-cavity Gunn-effect oscillator," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Microwave Aspects of Avalanche-Diode and Transferred Electron Devices*), vol. MTT-18, pp. 808–817, Nov. 1970.

Propagation Along Transversely Inhomogeneous Coaxial Transmission Lines

ROBERT E. MCINTOSH AND LUKE J. TURGEON

***Abstract*—Numerical "shooting" methods are employed in obtaining the dispersion curves of a coaxial waveguide loaded with a radially inhomogeneous dielectric. The utility of this technique is tested by comparing results with known analytical solutions. The method is also used to find the dispersion curves of a coaxial waveguide loaded with a radially Gaussian-distributed plasma.**

I. Introduction

Propagation of electromagnetic (EM) signals along transversely inhomogeneous transmission lines is of current interest because of the potential that these lines show for various waveguide applications. Various treatments of circular transmission lines having a radial inhomogeneity of the permittivity have been previously reported. Ah Sam and Klinger [1], for example, obtained a dispersion relationship for a coaxial line where the dielectric constant is inversely proportional to the square of the radius. Yamada and Watanabe have solved the wave equation for the azimuthally symmetrical circular waveguide where the dielectric constant varies quadratically with the radius [2]. Unfortunately, analytical solutions of the transmission properties of radially inhomogenous waveguides are unobtainable when the variation of the electrical properties of the medium is arbitrary. Much attention, therefore, has been given to numerical methods that employ finite-difference or variational techniques [3].

Numerical solutions have been obtained by various investigators for situations where the radially varying permittivity is approximated by a finite number N of concentric cylindrical shells [4], [5]. The dielectric constant of each of these shells is assumed to be constant, but not necessarily the same as the other shells. By assuming that the fields in each shell can be expressed in terms of Bessel functions, the problem is reduced to satisfying the boundary conditions at the inner

Manuscript received June 1, 1972; revised August 16, 1972. This work was supported in part by the Air Force Office of Scientific Research under Grant AFOSR G9-1722.
The authors are with the Department of Electrical Engineering, University of Massachusetts, Amherst, Mass. 01002.